8 PLAYING WITH CONSTRUCTIONS

8.1 Artwork

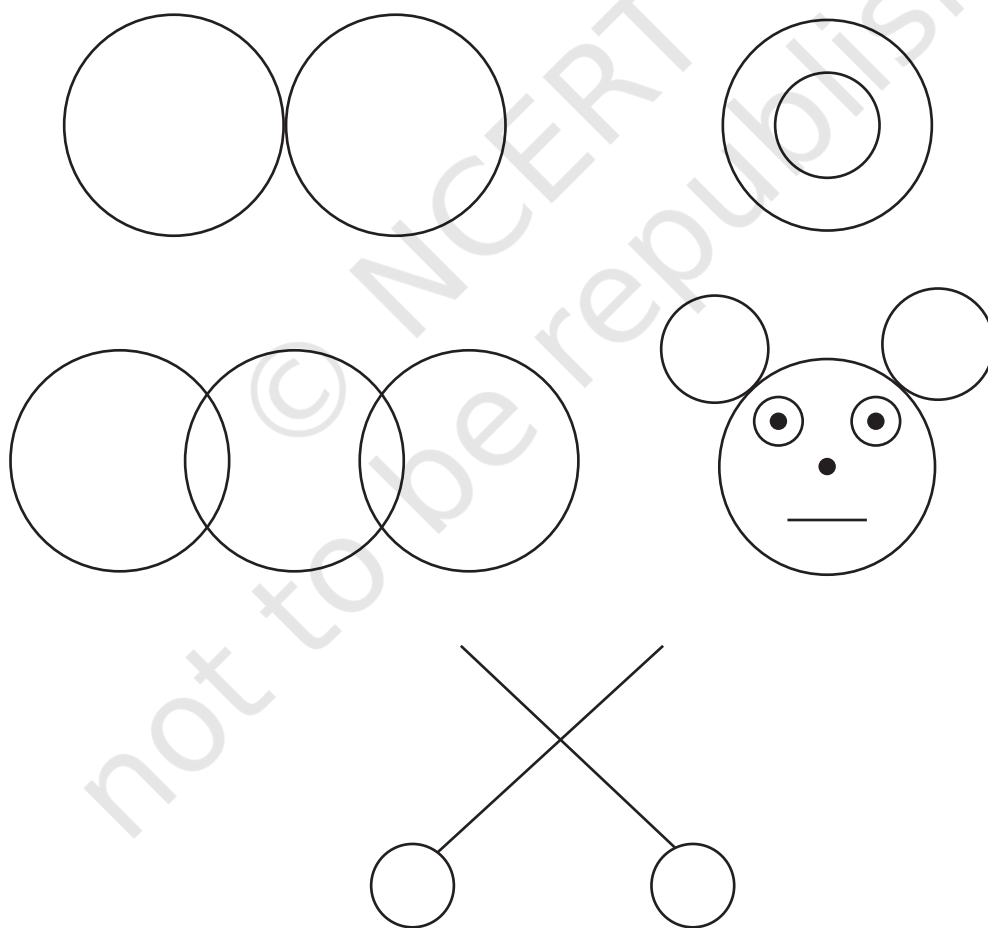Observe the following figures and try drawing them freehand.

Fig. 8.1

Now, arm yourself with a ruler and a compass. Let us explore if we can draw these figures with these tools and get familiar with a compass.

Observe the way a compass is made. What can one draw with the compass? Explore!

Do you know what curves are? They are any shapes that can be drawn on paper with a pencil, and include straight lines, circles and other figures as shown below:
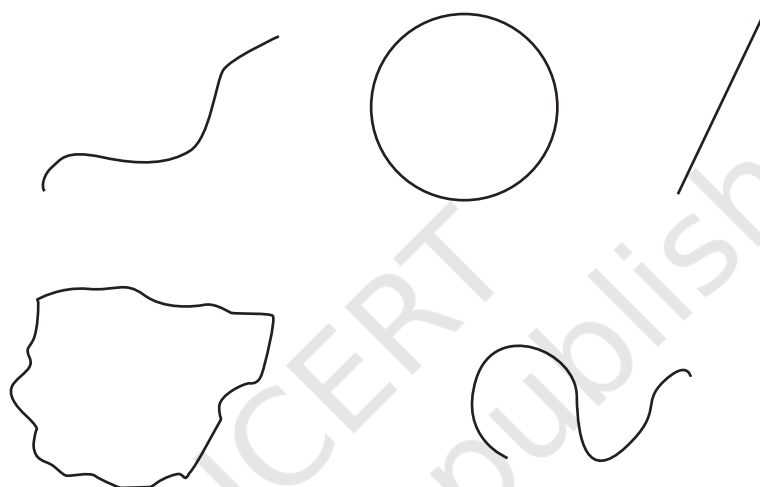
Mark a point 'P' in your notebook. Then, mark as many points as possible, in different directions, that are 4 cm away from P.

☀ *Think*: Imagine marking all the points of 4 cm distance from the point P. How would they look?

Try to draw it and verify if it is correct by taking some points on the curve and checking if their distances from P are indeed 4 cm.

Explore, if you have not already done so, and see if a compass can be used for this purpose.

You can start by marking a few points of distance 4 cm from P using the compass. How can this be done?
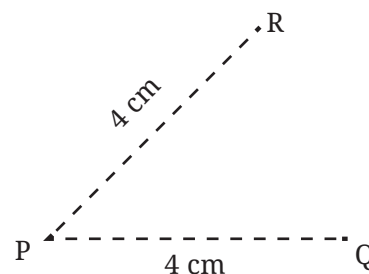
You will have to open up the compass against a ruler (see Fig. 8.2) such that the distance between the tip of the compass and the pencil is 4 cm.

❂   Now, try to get the full curve.

Hint: Keep the point of the compass fixed moving only the pencil.

What is the shape of the curve? It is a circle!

Take a point on the circle. What will be its distance from P—equal to 4 cm, less than 4 cm or greater than 4 cm? Similarly, what will be the distance between P and another point on the circle?

As shown in the figure, the point P is called the **centre** of the circle and the distance between the centre and any point on the circle is called the **radius** of the circle.
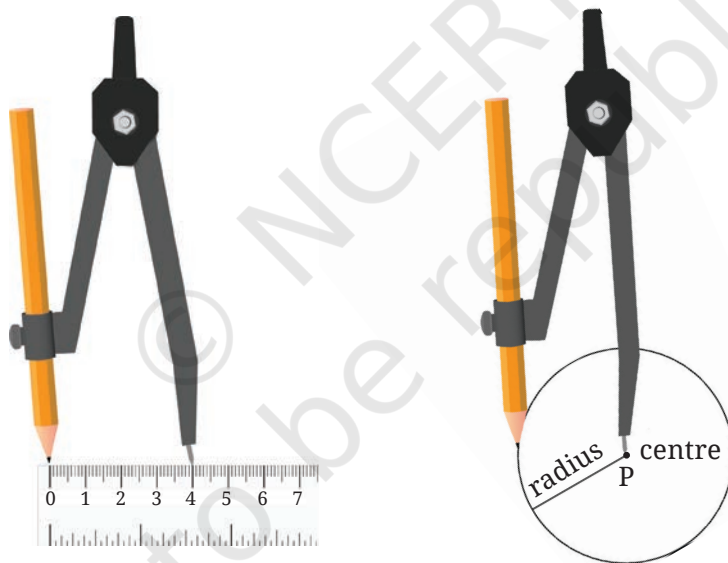
Fig. 8.2

Having explored the use of a compass, go ahead and recreate the images in Fig. 8.1.

Can you make the figures look as good as the figures shown there? Try again if you want to!
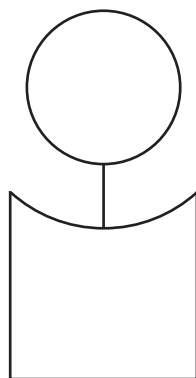
Also, has the use of instruments made the construction easier?

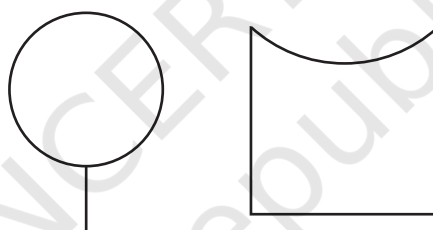Now try constructing the following figures.
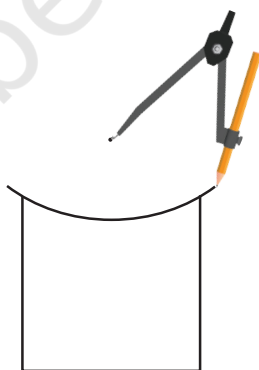
✸ Construct

1. A Person

How will you draw this?

This figure has two components.

You might have figured out a way of drawing the first part. For drawing the second part, see this.

The challenge here is to find out where to place the tip of the compass and the radius to be taken for drawing this curve. You can fix a radius in the compass and try placing the tip of the

compass in different locations to see which point works for getting the curve. Use your estimate where to keep the tip.

2. **Wavy Wave**

Construct this.
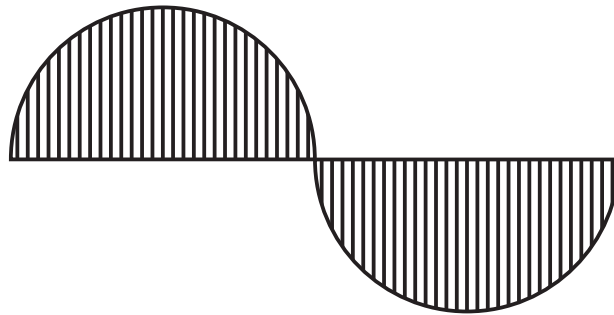
As the length of the central line is not specified, we can take it to be of any length.

Let us take AB to be the central line such that the length of AB is 8 cm. We write this as AB = 8 cm.

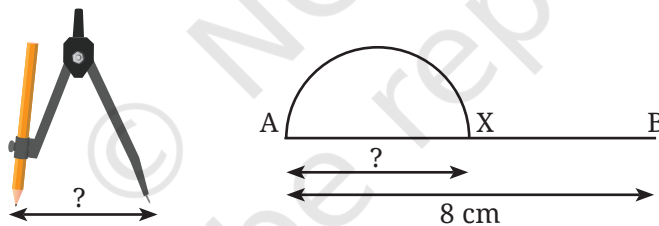Here, the first wave is drawn as a half circle.

✺ **Figure it Out**

1. What radius should be taken in the compass to get this half circle? What should be the length of AX?

2. Take a central line of a different length and try to draw the wave on it.

3. Try to recreate the figure where the waves are smaller than a half circle (as appearing in the neck of the figure, 'A Person'). The challenge here is to get both the waves to be identical. This may be tricky!

Try This

3. **Eyes**

How do you draw these eyes with a compass?

For a hint, go to the end of the chapter.

❂ Make other artwork of your choice with a ruler and a compass.

8.2 Squares and Rectangles

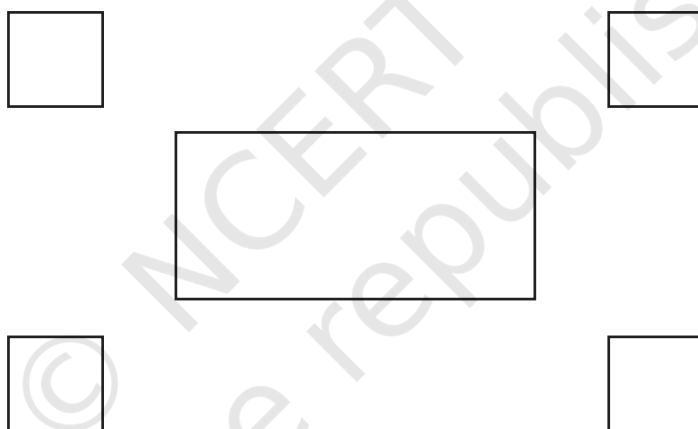Now, let us look at some basic figures having straight lines in their boundary.

Fig. 8.3

What shapes are these? Yes, these are our familiar squares and rectangles. But what makes them squares and rectangles?

Consider this rectangle ABCD.

The points A, B, C and D are the corners of the rectangle. Lines AB, BC, CD and DA are its sides. Its angles are ∠A, ∠B, ∠C and ∠D.

The blue sides AB and CD are called **opposite sides**, as they lie opposite to each other. Likewise, AD and BC is the other pair of opposite sides.
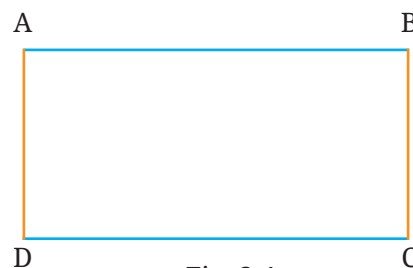

Fig. 8.4

Recall that, in a rectangle:

R1) The opposite sides are equal in length, and

R2) All the angles are 90°.

As in the case of rectangles, the corners and sides are defined for a square in the same manner.

A square satisfies the following two properties:

S1) All the sides are equal, and

S2) All the angles are 90°.

See the rectangle in Fig. 8.4 and the name given to it: ABCD. This rectangle can also be named in other ways—BCDA, CDAB, DABC, ADCB, DCBA, CBAD and BADC. So, can a rectangle be named using any combination of the labels around its corners? No! For example, it cannot be named ABDC or ACBD. Can you see what names are allowed and what names are not?

In a valid name, the corners occur in an order of travel around the rectangle, starting from any corner.
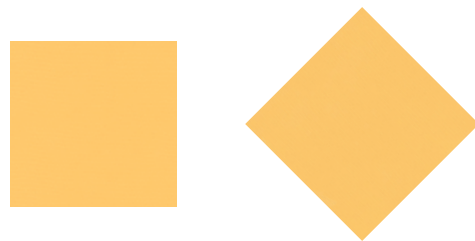
❈ Which of the following is not a name for this square?

1. PQSR
2. SPQR
3. RSPQ
4. QRSP
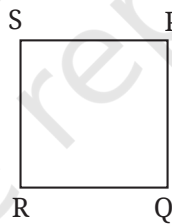
Rotated Squares and Rectangles

Here is a square piece of paper having all its sides equal in length and all angles equal to 90°. It is rotated as shown in the figure. Is it still a square?

Let us check if the rotated paper still satisfies the properties of a square.

- Are all the sides still equal? Yes.
- Are all the angles still 90°? Yes.

Rotating a square does not change its lengths and angles.

Therefore, this rotated figure satisfies both the properties of a square and so, it is a square.

By the same reasoning, a rotated rectangle is still a rectangle.
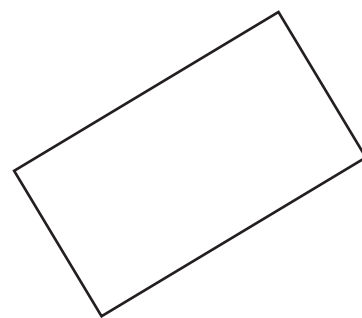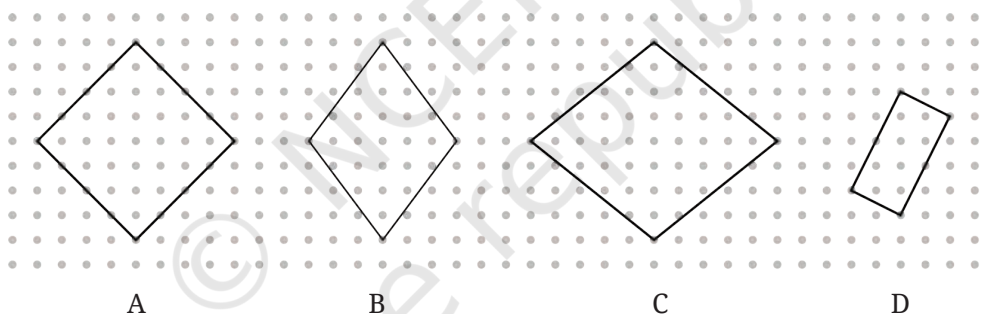
❈ Figure it Out

1. Draw the rectangle and four squares configuration (shown in Fig. 8.3) on a dot paper.

   What did you do to recreate this figure so that the four squares are placed symmetrically around the rectangle? Discuss with your classmates.

2. Identify if there are any squares in this collection. Use measurements if needed.

   A   B   C   D

   ❈ **Think:** Is it possible to reason out if the sides are equal or not, and if the angles are right or not without using any measuring instruments in the above figure? Can we do this by only looking at the position of corners in the dot grid?

3. Draw at least 3 rotated squares and rectangles on a dot grid. Draw them such that their corners are on the dots. Verify if the squares and rectangles that you have drawn satisfy their respective properties.
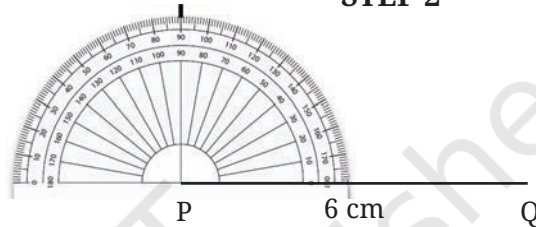
8.3 Constructing Squares and Rectangles

Now, let us start constructing squares and rectangles. How would you construct a square with a side of 6 cm?

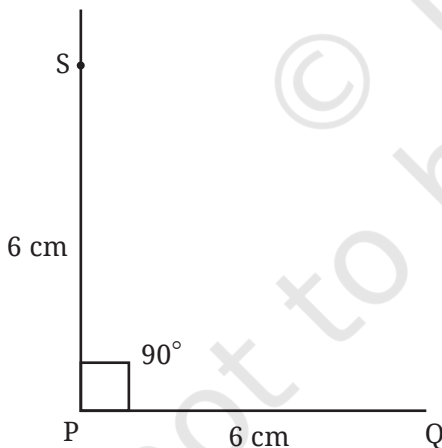For help, you can see the following figures. A square PQRS of side length 6 cm is constructed.

STEP 1

P ———— 6 cm ———— Q

STEP 2

P ———— 6 cm ———— Q
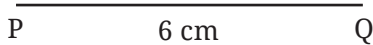
Mark a point to draw a perpendicular to PQ through P.
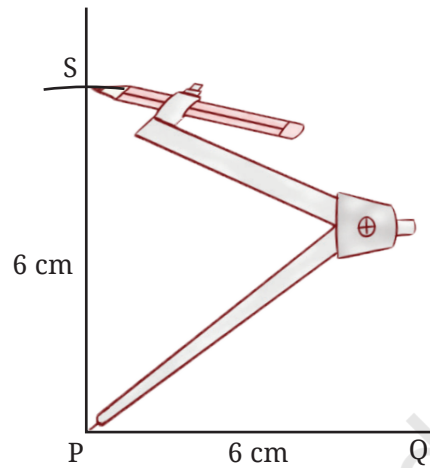
STEP 3
Method 1

S •

6 cm

90°

P ———— 6 cm ———— Q

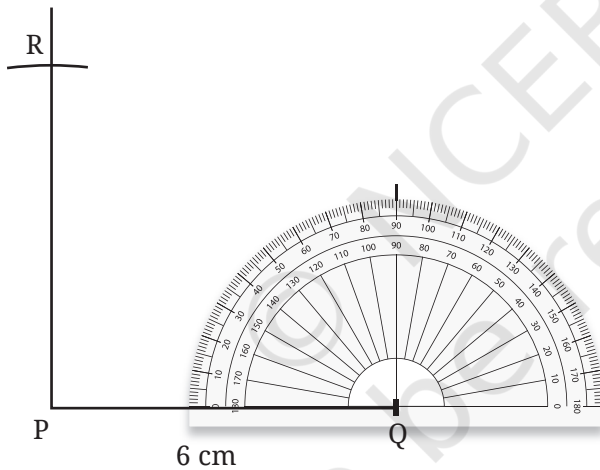Mark S on the perpendicular such that PS = 6 cm using a ruler.

Method 2

This can also be done using a compass.

Can you see why PS should be
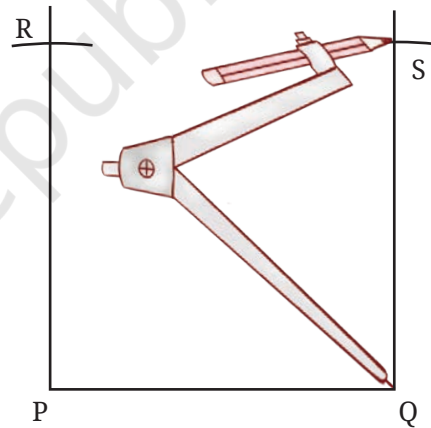6 cm long?

STEP 4
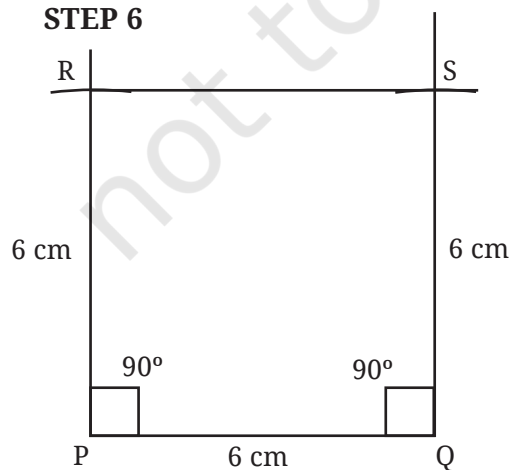Draw a perpendicular to line
segment PQ through Q.

STEP 5
If we had used the compass, then the next
point can easily be marked using it!

STEP 6

How long is the side RS and what
are the measures of ∠R and ∠S?

✹ Construct

1. Draw a rectangle with sides of length 4 cm and 6 cm. After drawing, check if it satisfies both the rectangle properties.

2. Draw a rectangle of sides 2 cm and 10 cm. After drawing, check if it satisfies both the rectangle properties.

3. Is it possible to construct a 4-sided figure in which—
   - all the angles are equal to 90º but
   - opposite sides are not equal?

Try This

8.4 An Exploration in Rectangles

Construct a rectangle ABCD with AB = 7 cm and BC = 4 cm.

Imagine X to be a point that can be moved anywhere along the side AD. Similarly, imagine Y to be a point that can be moved anywhere along the side BC. Note that X can also be placed on the end point A or D. Similarly, Y can also be placed on the end point B or C.
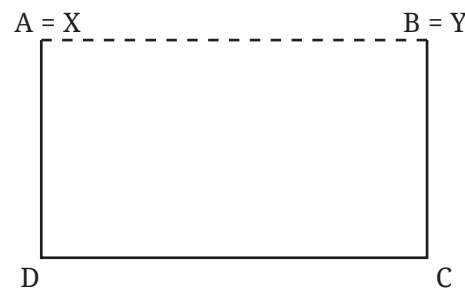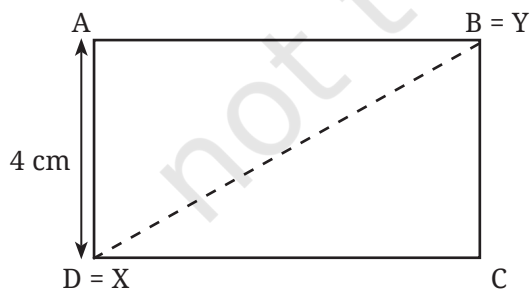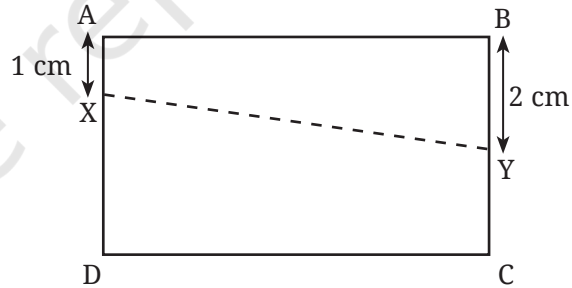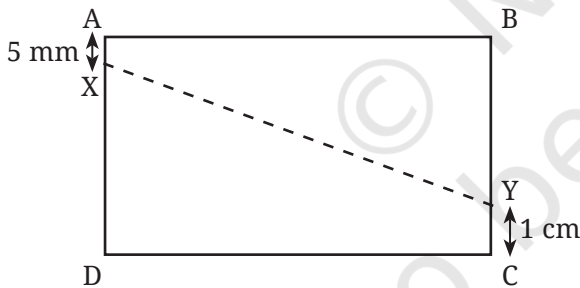
💮 At which positions will the points X and Y be at their closest? When do you think they will be the farthest? What does your intuition say? Discuss with your classmates.

Math Talk

Now, verify your guesses by placing the points X and Y on the sides and measure how near or far they are.

The distance between X and Y can be obtained by measuring the length of the line XY.

How does the minimum distance between the points X and Y compare to the length of AB?

Change the positions of X and Y to check if there are other positions where they are at their nearest or farthest. You could construct multiple copies of the rectangle and try out various positions of X and Y.

How will you keep track of the lengths XY for different positions of X and Y?

Here is one way of doing it. Suppose here are some of the positions of X and Y that you have considered:

- When X is 5 mm away from A and Y is 3 cm away from B,
  XY = ___ cm __ mm
- When X is 1 cm away from A and Y is 1 cm away from B,
  XY = ___ cm ___ mm
- When X is 2 cm away from A and Y is 4 cm away from B,
  XY = ___ cm ___ mm and so on.

💮 Is there *a* shorthand way of writing it down? In all the sentences, only the position of X, Y and the length XY changes. So we could write this as:

Distance of X from A	Distance of Y from B	Length of XY

❋ Have you checked what happens to the length XY when X and Y are placed at the same distance away from A and B, respectively? For example, as in the cases like these:

Distance of X from A	Distance of Y from B	Length of XY
5 mm	5 mm	
1 cm	1 cm	
1 cm 5 mm	1 cm 5 mm	

and so on.

❋ In each of these cases, observe
1. how the length XY compares to that of AB and
2. the shape of the 4-sided figure ABYX.

❋ How does the farthest distance between X and Y compare with the length of AC? BD?

❋ **Construct**

Breaking Rectangles

Construct a rectangle that can be divided into 3 identical squares as shown in the figure.
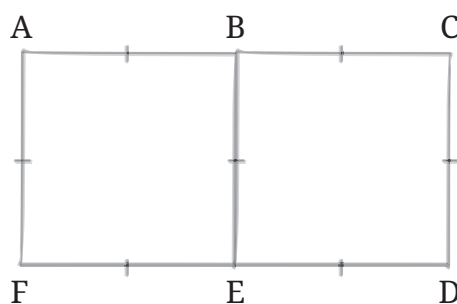
Solution

If this seem difficult, let us simplify the problem.

❋ *Explore*

What about constructing a rectangle that can be divided into two identical squares? Can you try it?

It is wise to first plan and then construct. But how do we plan? Can you think of a way?

One way is to visualise the final figure by drawing a rough diagram of it.

What can we infer from this figure?

Can you identify the equal sides?

Since, the two squares are identical,

AB = BC and FE = ED

Since ABEF and BCDE are squares, all the sides in each of the squares are equal. This is written as—

AF = AB = BE = FE

BE = BC = CD = ED

So, all the shorter lines are equal!

A convention is followed to represent equal sides. It is done by putting a '|' on the line. Refer to the rough figure.

Using this analysis, can you try constructing it? Remember, all that was asked for is a rectangle that can be divided into two identical squares and with no measurements imposed.

To draw the rectangle *ACDF*, one could assign any length to *AF*. For example, if we assign *AF* = 4 cm, then what must the length of *AC* be?

❂ *Explore*: Can the rectangle now be completed?

In fact, one could proceed by drawing AF without even measuring its length using a ruler. We could then construct a line perpendicular to AF that is long enough to contain the other side. As, AB = AF, we need to somehow transfer the length of AF

to get the point B. How do we do it without a ruler? Can it be done using a compass?

Observe, how the length of AF is measured using a compass.
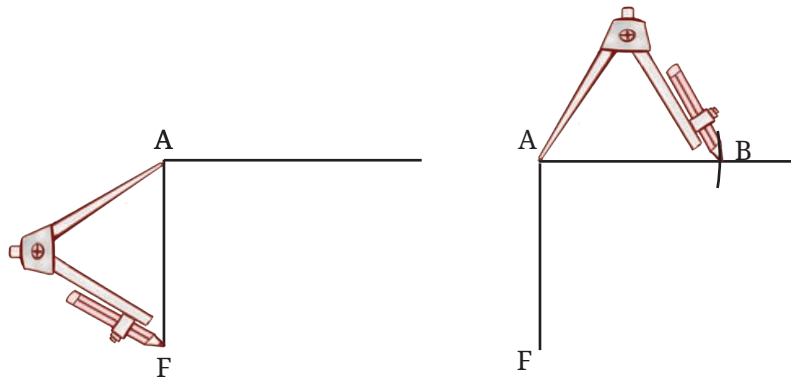
Use it to mark out the points B and C, and complete the rectangle.

❂ With this idea, try constructing a rectangle that can be divided into three identical squares.

❂ Give the lengths of the sides of a rectangle that cannot be divided into—

- two identical squares;
- three identical squares.

❂ **Construct**

1. **A Square within a Rectangle**

    Construct a rectangle of sides 8 cm and 4 cm. How will you construct a square inside, as shown in the figure, such that the centre of the square is the same as the centre of the rectangle?
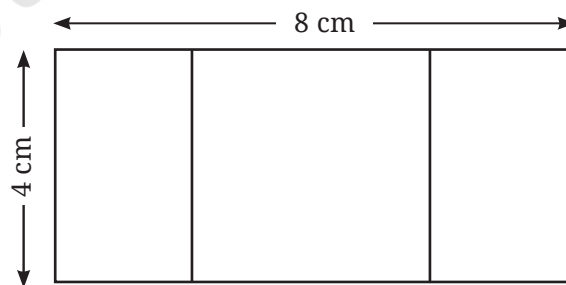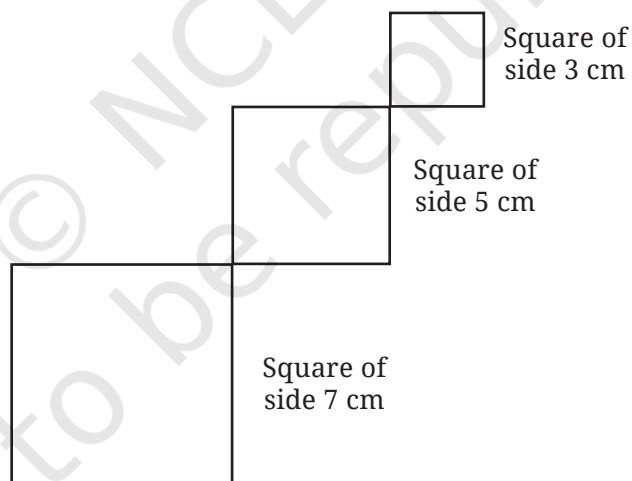
Hint: Draw a rough figure. What will be the sidelength of the square? What will be the distance between the corners of the square and the outer rectangle?
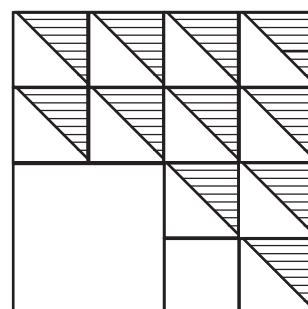
2. **Falling Squares**

4 cm

4 cm

4 cm

Each is a Square of side 4 cm
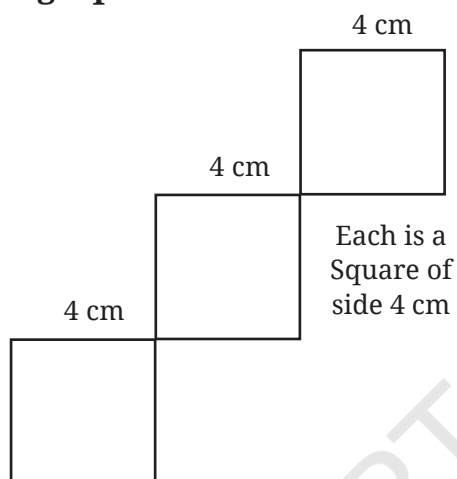
Make sure that the squares are aligned the way they are shown.

Now, try this.

Square of side 3 cm

Square of side 5 cm
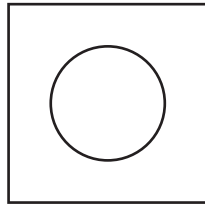
Square of side 7 cm

3. **Shadings**

Construct this. Choose measurements of your choice. Note that the larger 4-sided figure is a square and so are the smaller ones.
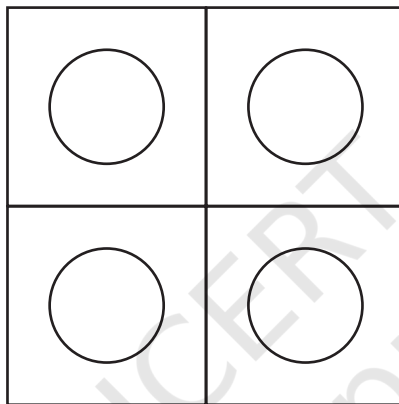
4. **Square with a Hole**

Observe that the circular hole is the same as the centre of the square.
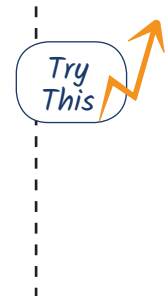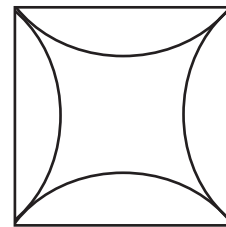
Hint: Think where the centre of the circle should be.

5. **Square with more Holes**

6. **Square with Curves**

This is a square with 8 cm sidelengths.

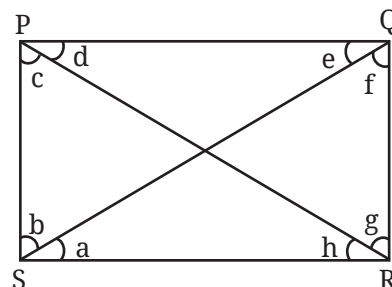*Hint:* Think where the tip of the compass can be placed to get all the 4 arcs to bulge uniformly from each of the sides. Try it out!

Try This

8.5 Exploring Diagonals of Rectangles and Squares

Consider a rectangle PQRS. Join PR and QS. These two lines are called the **diagonals** of the rectangle.

Compare the lengths of the diagonals. First predict the answer. Then construct a

rectangle marking the points as shown and measure the diagonals. In rectangle PQRS, the right angles at P and R are referred to as opposite angles. The other pair of opposite angles are the right angles at Q and S.

Observe that a diagonal divides each of the pair of opposite angles into two smaller angles. In the figure, the diagonal PR divides angle R into two smaller angles which we simply call g and h. The diagonal also divides angle P into c and d. Are g and h equal? Are c and d equal?

First predict the answers, and then measure the angles. What do you observe? Identify pairs of angles that are equal.

✷ *Explore*

How should the rectangle be constructed so that the diagonal divides the opposite angles into equal parts?

How will you record your observations? First, identify the parameters that need to be tracked. They are the sides of the rectangle and the 8 angles formed by the two diagonals. Are there any other measurements that you would want to keep track of?

Sides	A	B	C	D	E	F	G	H

In your experimentation, did you consider the case when all four sides of the rectangle are equal? That is, did you consider the case of a square? See what happens in this special case!

✷ What general laws did you observe with respect to the angles and sides? Try to frame and discuss them with your classmates.

> Math Talk

How can one be sure if the laws that you have observed will always be true?
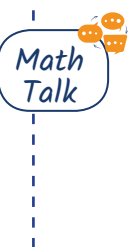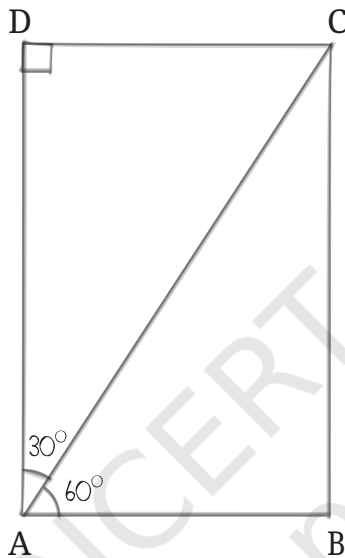
✹ Construct

1. Construct a rectangle in which one of the diagonals divides the opposite angles into 60° and 30°.
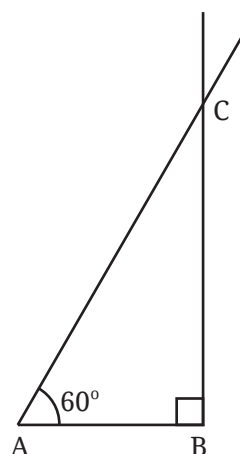
Solution

Let us start with a rough diagram.

In what order should its parts be drawn?
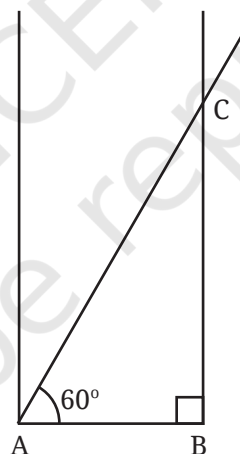
We will briefly sketch a possible order of construction.

STEP 1

AB is drawn with an arbitrary length. What is the next point that can be located?

STEP 2

STEP 3

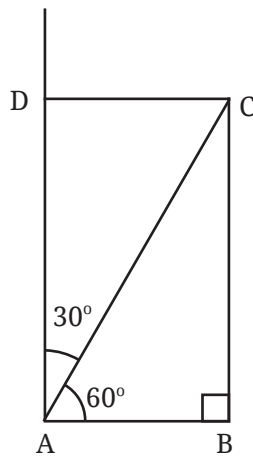We know the line on which D lies. Draw a line through A perpendicular to AB.

Now ∠A is divided into two angles. One measures 60°. Check what the other angle is.

There are at least two ways of finding the point D —

- One uses the fact that all the angles of a rectangle are right angles.
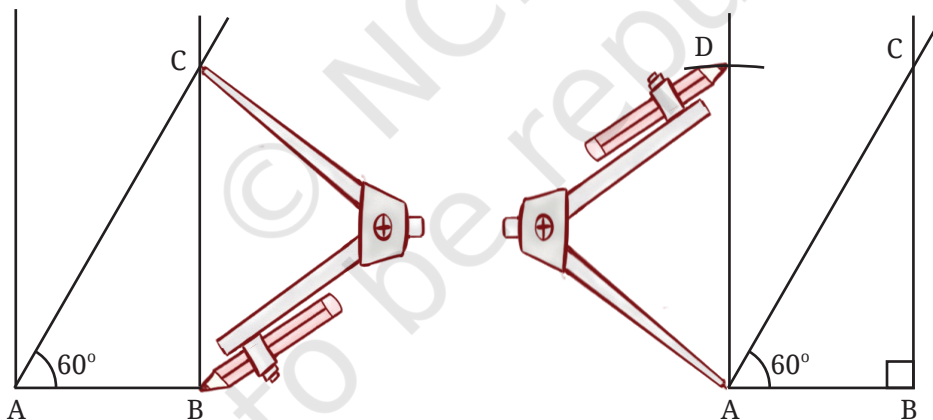- The other uses the fact that opposite sides are equal.

STEP 4
Method 1

Draw a line perpendicular to BC at C to get the point D.

Method 2

Using a compass, mark the point D such that AD = BC.
Join CD to get the required rectangle.

We have seen how to construct rectangles when their sides are given. But what do we do if a side and a diagonal is given?
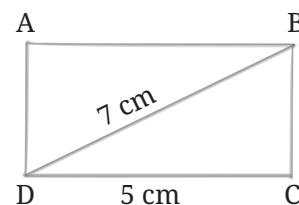
2. Construct a rectangle where one of its sides is 5 cm and the length of a diagonal is 7 cm.
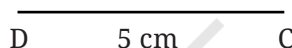
Solution

Let us draw a rough diagram.

Let us decide the steps of construction. Which line can be drawn first?

STEP 1

The base CD measuring length 5 cm can be easily constructed.

D   5 cm   C

Next?

STEP 2

Draw a perpendicular to line DC at the point C. Let us call this line *l*.

D   5 cm   C
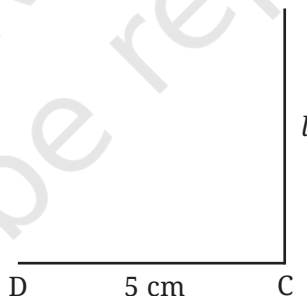
This is easy as we know that this line is perpendicular to the base. The point B should be somewhere on this line *l*.

❂ How do we spot it? What else do we know about the position of B?

We know that it is at a distance of 7 cm from the point D.

One of the ways of marking B is by taking a ruler and trying to move it around to get a point on line *l* that is 7 cm from point D. However, this requires trial and error. There is another efficient method which doesn't involve trial and error.

For this, instead of trying to get that one required point of distance 7 cm from D, let us explore a way of getting all the points of distance 7 cm from D.

We know what this shape is!

STEP 3
Method 1
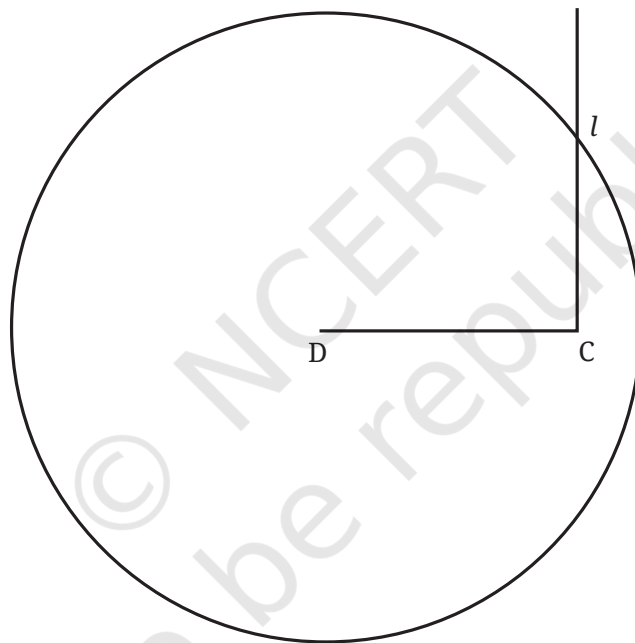
Construct a circle of radius 7 cm with point D as the centre.

Can you spot the point B here? Remember that it is 7 cm away from point D and on the line *l*.

Consider the point at which the circle and the line intersect. What is its distance from point D? If needed, check your figure. What do you observe?
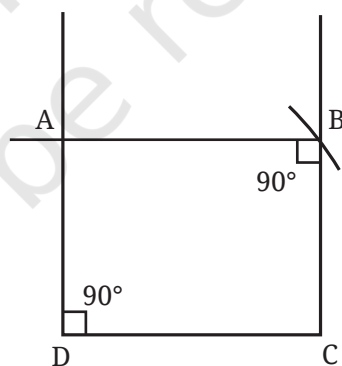
The point where the circle intersects the line *l* is the required point B.

Method 2

To locate the point B, was it necessary to draw the entire circle? We can see that only the arc near the line *l* is needed. So, the third step can also be done as shown in the figure below.
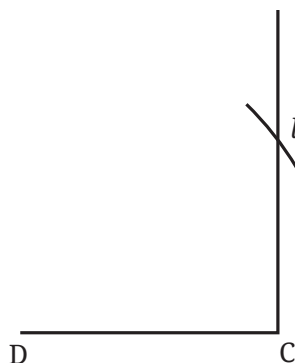
Having marked the three points of the rectangle, we only need to complete it. Recall that we were in a similar situation in the previous problem also. We saw two methods of completing the rectangle from here. We could follow any one of those methods.

STEP 4

Construct perpendiculars to DC and BC passing through D and B, respectively. The point where these lines intersect is the fourth point A.

Check if ABCD is indeed a rectangle satisfying properties R1 and R2.

Construct

1. Construct a rectangle in which one of the diagonals divides the opposite angles into 50° and 40°.

2. Construct a rectangle in which one of the diagonals divides the opposite angles into 45° and 45°. What do you observe about the sides?

3. Construct a rectangle one of whose sides is 4 cm and the diagonal is of length 8 cm.

4. Construct a rectangle one of whose sides is 3 cm and the diagonal is of length 7 cm.

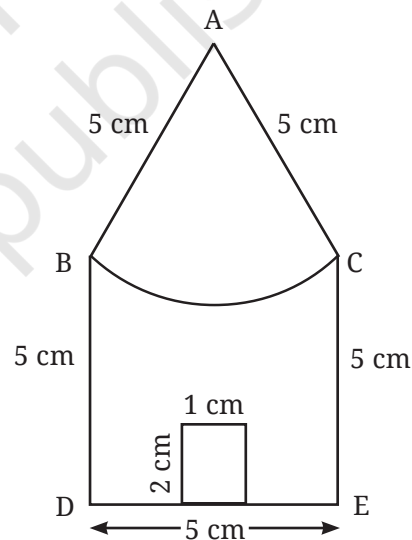# 8.6 Points Equidistant from Two Given Points
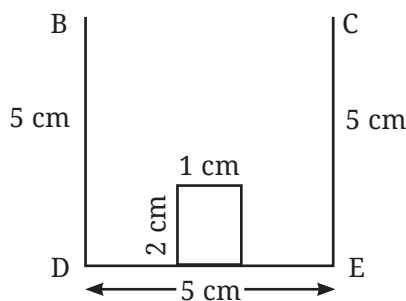
Construct

House

Recreate this figure.

Note that all the lines forming the border of the house are of length 5 cm.

Solution

The first task is to identify in what sequence the lines and curve will have to be drawn.

STEP 1

Can you complete the figure? Try!

We need to locate the point A that is of distance 5 cm from the points B and C. You might have realised that this can be done using a ruler. However, this leads to a lot of trial and error. This construction can be further simplified. How?

If you have guessed that this can be done by the use of compass, you are right! Go ahead and explore how the point A can be located without trial and error.

There is a similarity between the problem of finding point A in this problem and point B at step 3 of the second solved example of the previous section (see page 209).

STEP 2
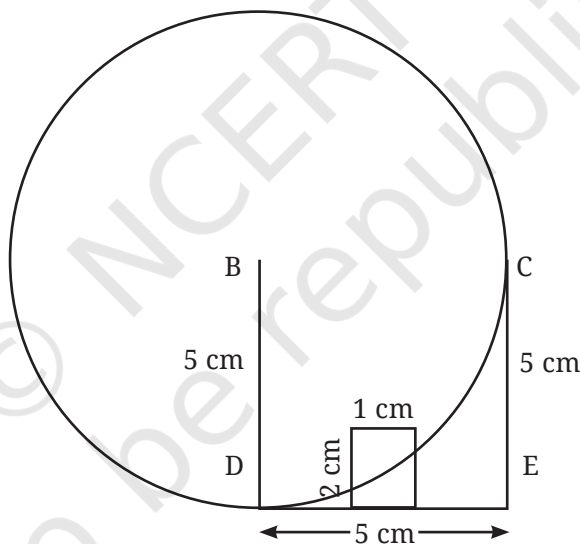
Draw a curve that has all its points of 5 cm from the point B; the circle centred at B should be with 5 cm radius.

Does this help in spotting the point A? Construct and explore in the figure.

The point A can be located by finding the correct point on the circle that is of distance 5 cm from the point C. Again, this can be done using a ruler. But can we use a compass for this?

STEP 3
Method 1

Take a radius of 5 cm in the compass and with C as the centre, draw a circle.
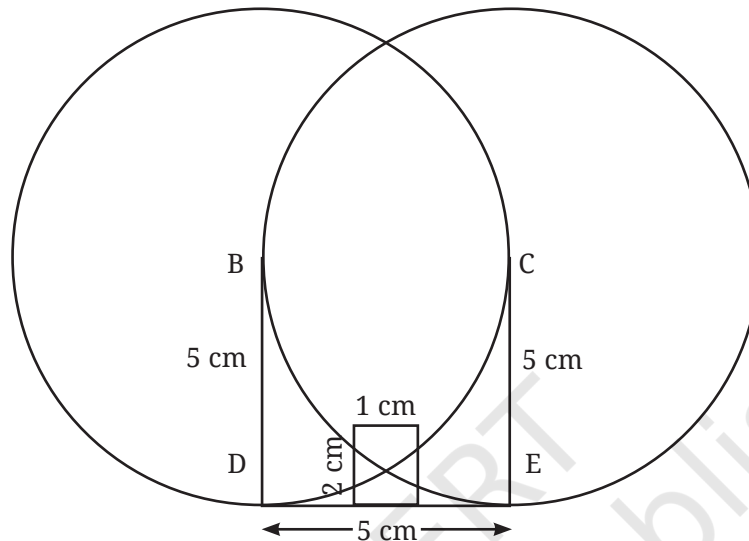
Are you able to spot the point A? Check the figure in your notebook. What do you observe?

See the point at which both the circles intersect. How far is it from the point B?

How far is it from C?

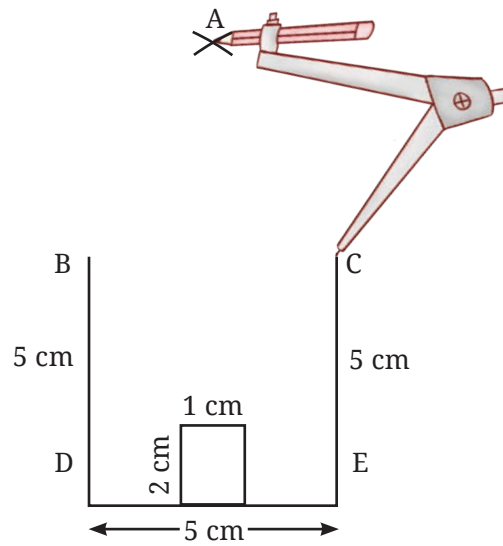Thus, this is the point A!

✸ *Think*

Was it necessary to draw two full circles to get the point A? We only needed part of both the circles.

Method 2

So the point A could have been obtained just by drawing arcs of radius 5 cm from points B and C.
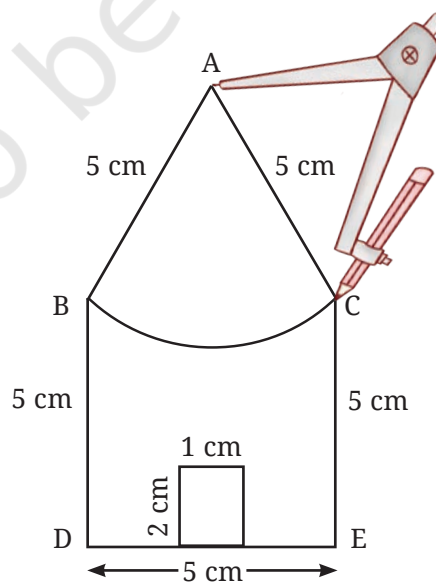
Join A to B and A to C by straight lines.

Having obtained point A, what remains is the construction of the remaining arc. How do we do it?

Can we use the fact that A is of distance 5 cm from both B and C?

STEP 4

Take 5 cm radius in the compass and from A, draw the arc touching B and C as shown in the figure.


The house is ready!

✿ Construct

1. Construct a bigger house in which all the sides are of length 7 cm.

2. Try to recreate 'A Person', 'Wavy Wave', and 'Eyes' from the section 'Artwork', using ideas involved in the 'House' construction.

3. Is there a 4-sided figure in which all the sides are equal in length but is not a square? If such a figure exists, can you construct it?

Hints

A) Eyes (from 8.1 Artwork and Construct above (page no. 215).

Part of the construction is shown earlier. Observe it carefully. You will see two horizontal lines drawn lightly. In geometric constructions, one often constructs supporting curves or figures that are not part of the given figure but help in constructing it.
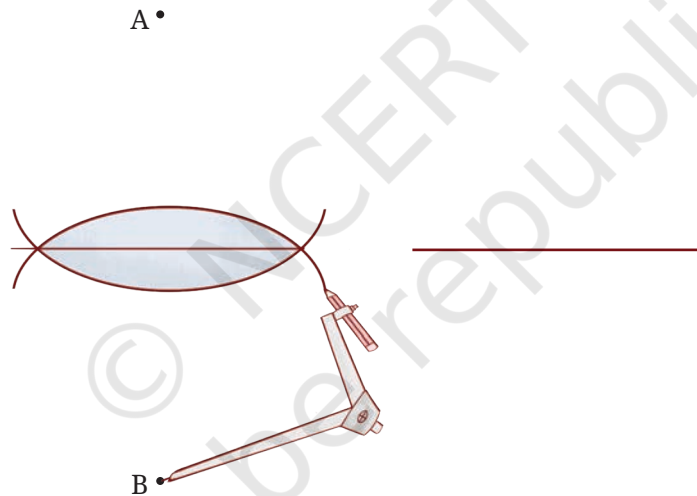

The technique to draw the upper and the lower curves of the eye is the same as that used in the figure, 'A Person'. Points A and B are the locations where the tip of the compass is placed when drawing the curves of the eye. Note that the upper curve and the lower curve should together form a symmetrical figure. For this to happen, where should these points A and B be placed? Make a good estimate.

Try to get the eyes as symmetrical and identical as possible. This might need many trials.

B) (From Construct above (page no. 211).

For the purpose of construction, let us take the side lengths to be of 5 cm. Consider this figure.
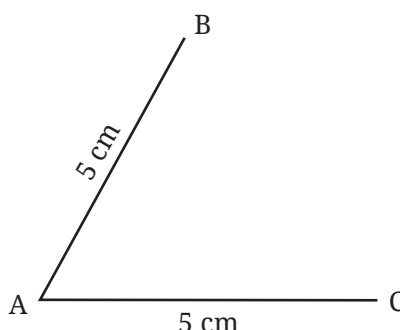
We need to identify only one more point to make this a 4-sided figure. That point, let us call it D, should be 5 cm from both B and C. How can such a point be found?

Can any of the ideas used in the 'House' problem be used here?

SUMMARY

■ All the points of a circle are at the same distance from its **centre**. This distance is called the **radius** of the circle.

■ A compass can be used to construct circles and their parts.

■ A rough diagram can be useful in planning how to construct a given figure.

■ A rectangle can be constructed given the lengths of its sides or that of one of its sides and a diagonal.